# Chapter 6

# Application of a Dedicated Stochastic Non-linear Dynamic Time Domain Analysis Program in Design and Assessment of Jack-ups

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## INTRODUCTION

Over the last 4 years the jack–up industry has developed an increased interest in site specific jack–up assessments and consequently in jack–up analysis methods. In this context, MSC has been working on the development, testing and verification of the computer program SIMSEP, providing a stochastic, non–linear, dynamic, structural analysis in the time domain, dedicated to jack–ups.

The main purpose of this paper is to describe and discuss our experience with the application of this program and more in particular the effect a new and different analysis technique may have on the actual design.

## THE PROGRAM

The theoretical background and main features of SIMSEP are described in ref [1]. As a result of verification and application of the program it was found that the originally implemented MPM (Most Probable Maximum of a particular response parameter) estimation method was not adequate.

Presently we are using the following procedure to determine an MPM estimate:

- the program is used to produce 10 independent simulations of a 3 hours storm characterized by a significant wave height with an appropriate return period (i.e 50 years), each yielding a 3 hrs maximum for the response parameters of interest
- a Gumbel distribution is fitted through each set of 10 maxima on the basis of the maximum likelihood method
- from these distributions the MPM is calculated as the value corresponding to the 0.37 quantile, thus producing an MPM with an expected recurrence of once in a 3 hours storm

This means that the MPM is based on simulations to a total of 30 hours real time.

## DESIGN APPLICATION

MSC is working on the development of a jack–up design designated for multi–purpose operations in the Southern North Sea. For a general arrangement of this jack–up for offshore accommodation, maintenance and construction, see figure 1.

In outline the design has the following main particulars:

| | | |
|---|---|---|
| Length of hull | 50 | m |
| Width of hull | 50 | m |
| Hull depth | 6 | m |
| Number of legs | 3 | |
| Leg length | 82 | m |
| Elevated weight | 5600 | ton |
| Accommodation | $150^+$ | men |
| Elevating system | MSC hydraulic | |
| Crane | 300 | ton at 30 m |
| Helideck for | S61N | |
| Free deck area | 1000 | $m^2$ |
| Payload | 1600 | ton |

Table 1.:
Main particulars of jack–up.

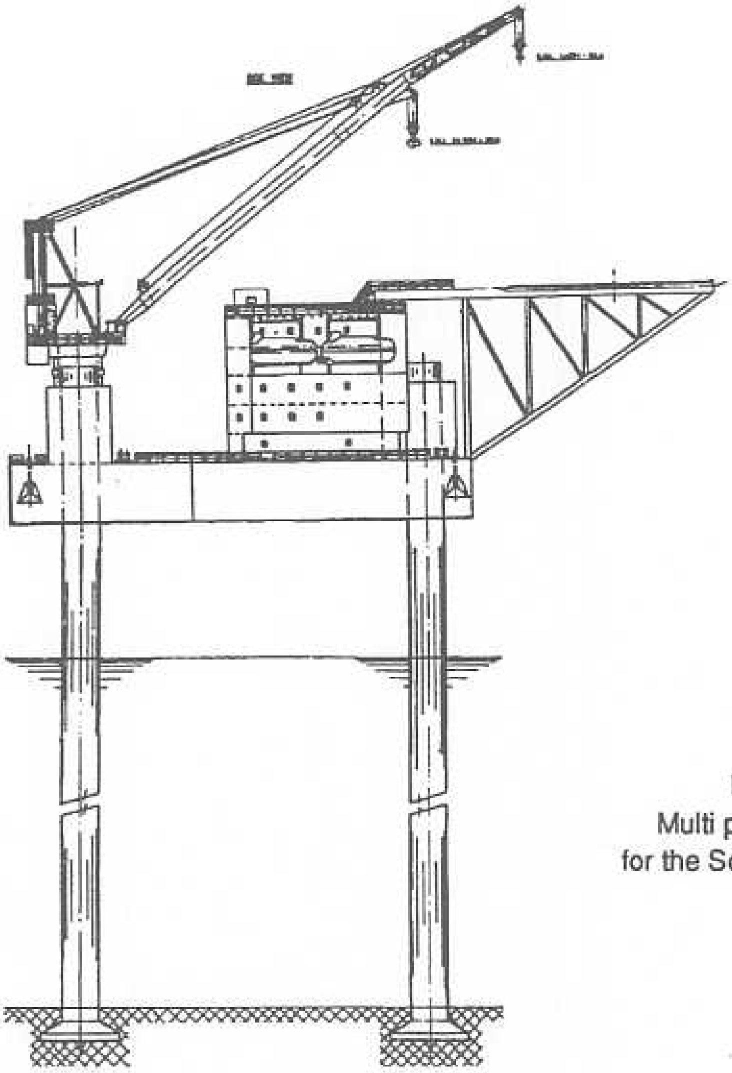


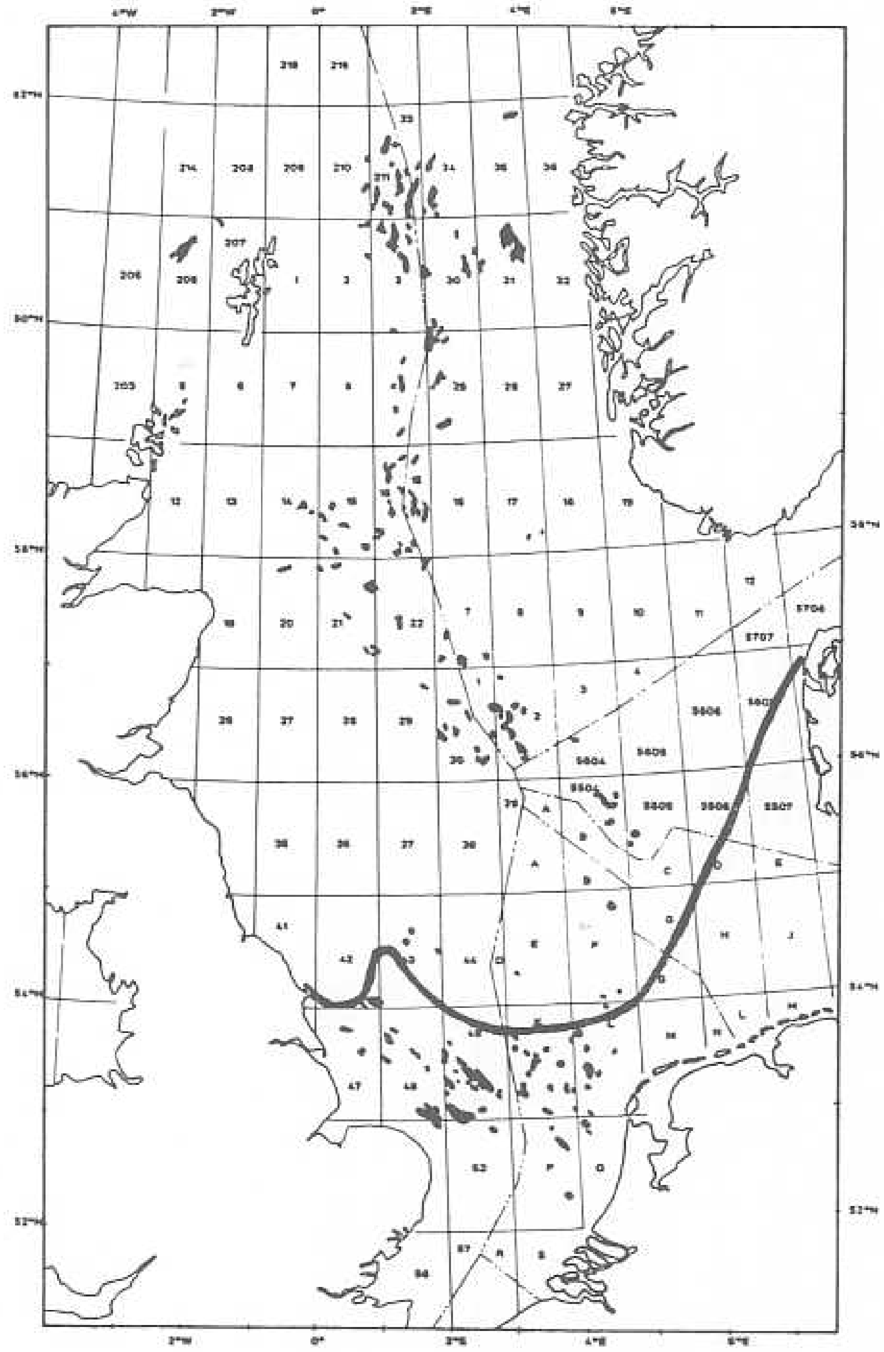
Figure 1.:
Multi purpose jack–up
for the Southern North Sea.

The design criteria for this jack–up are defined as follows:

| | | |
|---|---|---|
| Waterdepth | 45 | m |
| Max. wave height | 18.5 | m |
| Ass. period | 14 | sec |
| Wind velocity | 43 | m/sec |
| Current velocity | 1.5 | m/sec |
| Current profile | 1/7 | |

Table 2.:
Environmental design criteria.

These criteria lead to an operational area as shown in figure 2.



*Operational area is south of the line at approx $54^0$N*

Figure 2.:
Operational area of the
multi purpose jack–up.

In the design process we developed 2 alternatives:

- the jack–up equipped with cylindrical legs (figure 3.a)
- the jack–up equipped with triangular truss type legs (figure 3.b)

## LEGLOAD (QUASI–STATIC) ANALYSIS

Both alternatives are initially analyzed using our standard jack–up analysis LEGLOAD. This program uses a deterministic quasi–static analysis properly accounting for non–linear effects such as the P–delta effect. Dynamics are included via a single mass spring analogy by a DAF on the horizontal deflection of the hull. The dynamic contribution to the structural parameters is found through the application of an 'inertial load' at hull level of a magnitude required to achieve the amplified horizontal deflection.

Input data for this analysis are as follows:

| | | *circular leg* | *truss leg* |
|---|---|---|---|
| Legs in equilateral triangle with sides of | m | 43 | 43 |
| Elevated weight | MN | 55 | 55 |
| Weight of leg | kN/m | 64 | 44 |
| Buoyancy of leg | kN/m | 8.0 | 8.4 |
| Pin point support at penetration of | m | 4.0 | 4.0 |
| Waterdepth | m | 45 | 45 |
| Airgap | m | 15.0 | 15.0 |
| Rotation stiffness leg to hull | MNm/deg | 1283 | 1283 |
| Specific diam. of leg | m | 3.9 | 6.0 |
| CD x D of leg | m | 2.73 | 2.4 |
| CM x $D^2$ of leg | $m^2$ | 30.4 | 3.24 |
| Leg added mass | ton/m | 24 | 2.0 |
| Effective leg stiffness | $m^4$ | 1.4 | 1.4 |

Table 3.:
Structural data for overall analysis.

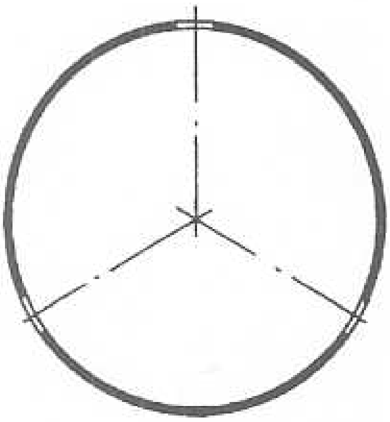


Figure 3.a.:
Circular leg.

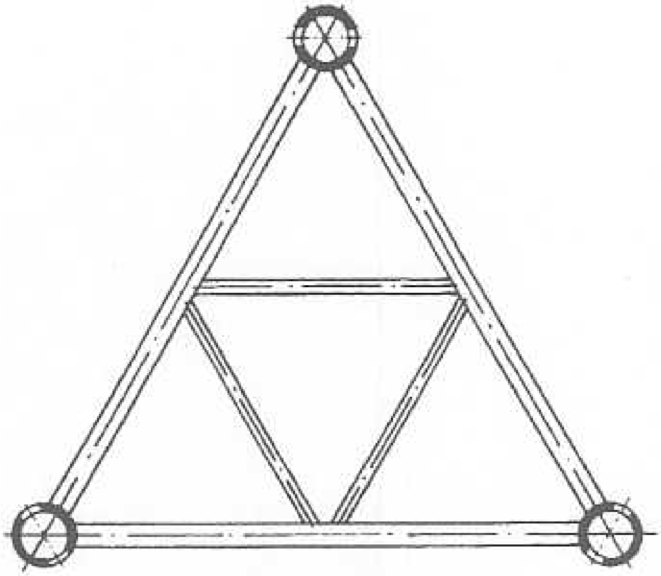


Figure 3.b.:
Truss leg.

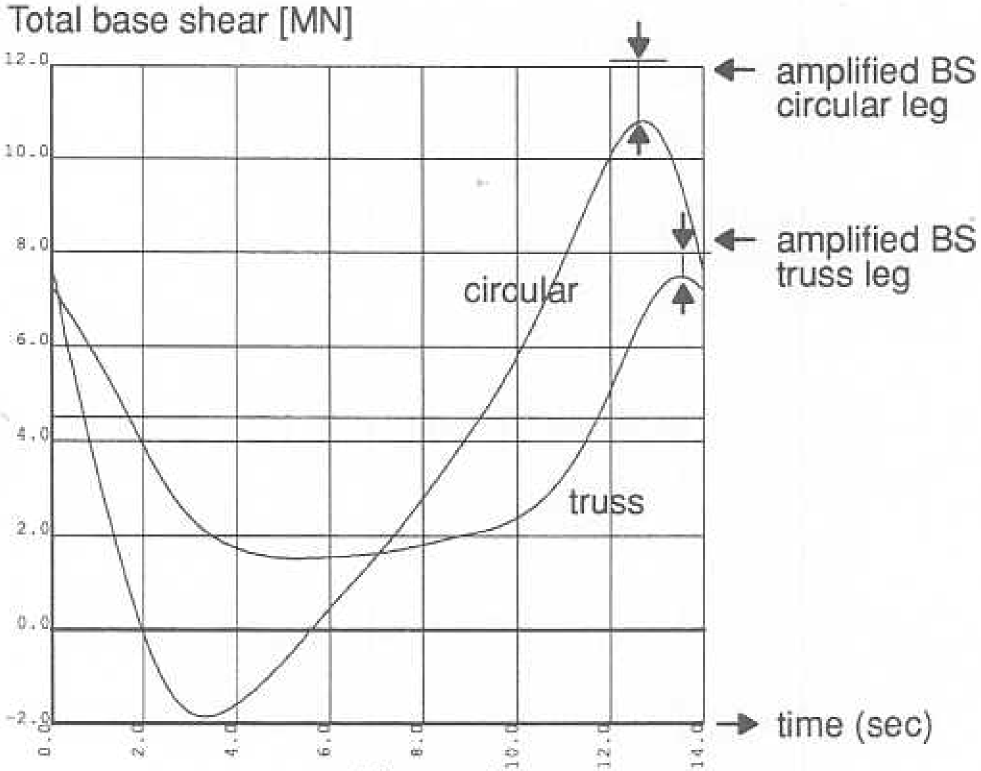


Figure 4.:
Base shear in one wave cycle.

As can be seen both alternatives are structurally (leg stiffness, hull weight, free leg length) equivalent. The hydrodynamic (Morison) coefficients show an essential difference. The circular leg has a drag coefficient (CD x D) 1.14 times and an inertia coefficient (CM x $D^2$) of even 12 times higher than the truss type leg. The significantly higher leg added mass results in a higher first natural period for the circular leg.

The results of the analysis are as follows:

| | | *circular leg* | *truss leg* |
|---|---|---|---|
| First natural period | sec | 6.85 | 6.18 |
| Hull deflection | m | 1.42 | 0.96 |
| Overturning moment | MNm | 546.8 | 383.6 |
| Base shear | MN | 12.0 | 7.9 |
| Max. leg moment | MNm | 221 | 157.3 |
| Max. vertical footing reaction | MN | 41.1 | 34.1 |
| Min. safety against overturning | – | 1.57 | 2.02 |
| Min. ratio vertical/horiz. footing reaction | – | 2.1 | 4.02 |
| Required preload ballast | MN | 76.7 | 56.4 |
| Required preload reaction in jacking system | MN | 43.9 | 37.1 |
| DAF on deflection | – | 1.31 | 1.24 |

Table 4.:
Results from LEGLOAD analysis.

The results show that, as one can expect, the maximum wave current forces on the jack–up are approx 50% higher for the circular leg alternative.

Additionally the range from the minimum to maximum wave current forces on the circular leg unit is approx twice the same range for the truss type unit (see figure 4). The horizontal deflection of both units shows the same behaviour.

A consequence of this and the DAF method, which is related to the amplitude of the horizontal deflection, is that although the DAF's are approx equal, the final 'inertial load' for the circular leg is approx twice the inertia load for the truss type leg unit.

All this results in a significantly higher structural loading for the cylindrical leg unit. In the design this has consequences for the steel scantlings of legs and hull, jacking (preloading) capability, and preload ballast capacity.

Next to these structural considerations there is of course the economics of the design. The circular leg version clearly requires more steel to build, on the other hand unit manufacturing cost for a truss leg are higher than for a circular leg.

A quick and simple cost difference analysis on the basis of rough unit cost estimates shows the following:

| | *circular leg* | | *truss leg* | |
|---|---|---|---|---|
| | *ton* | *cost Dfl x $10^6$* | *ton* | *cost Dfl x $10^6$* |
| Legs (excl. footing) | 1605 | 9.6 | 1095 | 15.3 |
| Footings (extra) | 50 | 0.6 | – | – |
| Jackhouses (extra) | 60 | 0.6 | – | – |
| Hull (extra) | 165 | 1.0 | – | – |
| Jacking system (extra) | – | 0.6 | – | –.– |
| Totals | 1880 | 12.4 | 1095 | 15.3 |
| Extra weight | 785 | – | – | – |
| Extra cost | – | – | – | 2.9 |

Table 5.:
Cost difference based on LEGLOAD analysis.

Clearly the advantage in manufacturing cost for the circular leg offsets the additional cost in leg footing, jacking systems and hull steel.

The conclusion from this analysis is therefore simple: the jack–up equipped with cylindrical legs is attracting higher loads and requires more steel, but is complying with all requirements and is approx Dfl 3 million cheaper than the truss type leg alternative.

This conclusion is of course very much a function of price levels for labour and material but could perhaps also be dependent on the type and rigorousness of the overall analysis. To investigate this, the structural analysis is repeated with our latest time domain analysis program SIMSEP.

## SIMSEP (TIME DOMAIN) ANALYSIS

In comparison with the deterministic quasi–static analysis the method in this program is doing away with a multitude of assumptions and approximations:

- the quasi–static maximum design wave is replaced by a irregular wave condition determined by a significant wave height and appropriate zero–up crossing period
- the assumption that the position of the design wave relative to the jack–up giving a maximum overturning moment is also giving a maximum for all parameters of interest
- all non–linear effects in loading (drag forces, relative velocity and variation in water level), geometry (P–Δ), structure (hull to leg interface) and foundation can be taken into account properly
- the dynamic response is fully and properly accounted for

The drawback of this time domain analysis is that, by the very nature of the method, it does not yield a single maximum value for a design parameter as the deterministic, quasi–static method does (or is assumed to do). The time domain analysis results in a registration of the parameters as it varies in time (see figure 5).

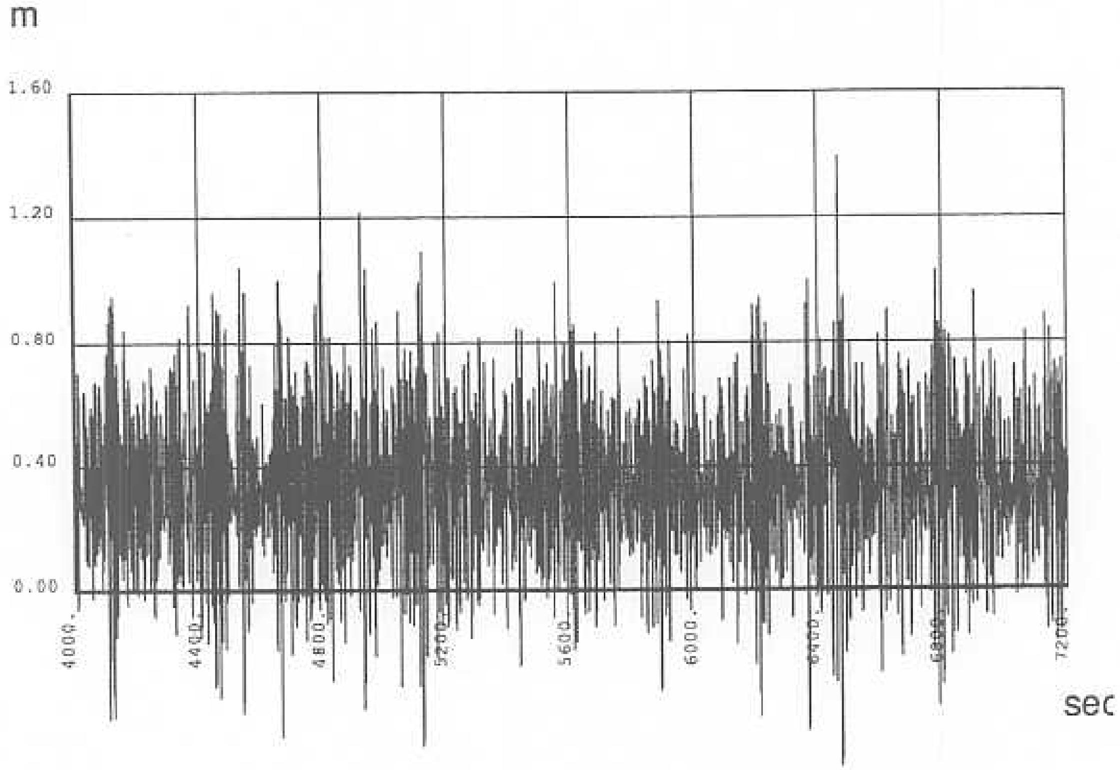


Figure 5.:
Time registration of hull deflection for the circular leg.

What we need for the further design process is a single value to be able to determine steel scantlings or equipment capacities. This value is calculated from the available registrations following a procedure as outlined above. This MPM value is determined on the basis of the same recurrence as the maximum design wave for the deterministic quasi–static analysis method (i.e once within a 3 hours storm).

Similar to the analysis with LEGLOAD, SIMSEP has been applied to both alternatives with wave, current and wind action in two directions to determine the maximum vertical leg reaction (wave direction in symmetry plane towards one leg) and minimum overturning safety (the opposite direction).

The structural model used in SIMSEP has been kept as similar as possible to the model used in LEGLOAD.

The mass models used in SIMSEP and LEGLOAD, however, are not similar. The SIMSEP model is having mass lumps along the leg stick models and in the hull, while the LEGLOAD single mass–spring model to determine DAF assumes one mass representing the elevated hull mass and an appropriate part of the leg mass. Both mass models, however, have been tuned to provide the same first natural periods.

A very distinct difference for the SIMSEP analysis is the input value for hydrodynamic coefficients:

| | | *circular leg* | *truss leg* |
|---|---|---|---|
| CD x D | – | 3.92 | 3.4 |
| CM x $D^2$ | – | 30.4 | 3.24 |

Table 6.:
Hydrodynamic coefficients for time domain analysis.

The inertia coefficients remain unchanged but the drag coefficients are increased with a factor 1.4 for both leg types. This increase is based on the notion that the non–linear single wave theories as used in the quasi–static method yield conservative (higher) wave kinematics which justify a CD at 0.7 level, where as the random wave definition is closer to reality and therefore require a CD at 1.0 level.

In order to obtain reliable MPM–values of the response parameters of interest according to the statistical procedure described above, for each wave direction, 10 separate 3 hrs simulations have been performed. For both leg types the same kinematical time histories have been used for the same wave directions, thus ruling out incorrect comparison of both rig types due to stochasticity of the hydrodynamic loading.

The wave/current kinematical time histories have been generated prior to the actual simulations:

- for a unidirectional Pierson–Moskowitz spectrum, $H_s = 10$ m, $T_{zup} = 11.0$ sec
- using Inverse Fast Fourier Transform techniques (over 16000 wave frequency components)
- using Wheeler stretching in wave crests, no stretching in troughs
- without current flow correction (surface current velocity constant with time)

The actual simulations have been performed with:

- a time step of 0.2 s
- 2% of critical damping
- inclusion of fluid structure interaction
- all relevant non–linearities like P–Δ effect, variable leg submergence, etc

The procedure results in a total of 540.000 data points (30 hours) for each response parameter. Figures 6 and 7 show response spectra for the OTM for both leg types.

From the statistical post–processing of these response time histories the following MPM–values have been found:

| | | *circular leg* | *% ** | *truss leg* | *% ** |
|---|---|---|---|---|---|
| First natural period | sec | 6.85 | | 6.18 | |
| Hull deflection | m | 1.31 | *92.2* | 1.06 | *110.4* |
| Overturning moment | MNm | 529 | *96.7* | 438 | *114.2* |
| Base shear | MN | 10.9 | *90.8* | 8.6 | *108.9* |
| Max. leg moment | MNm | 196 | *88.7* | 160 | *101.7* |
| Max. vertical footing reaction | MN | 39.9 | *97.1* | 35.6 | *104.4* |
| Min. safety against overturning | – | 1.51 | *96.2* | 1.80 | *89.1* |
| Min. ratio vertical/ horizontal footing reaction | – | 1.91 | *90.9* | 3.22 | *80.1* |
| Required preload ballast | MN | 72.5 | *94.5* | 61.9 | *109.8* |
| Required preload reaction in jacking system | MN | 42.5 | *96.8* | 39.0 | *105.1* |

*) = results from quasi–static analysis (table 4) are 100%

Table 7.:
Results from SIMSEP analysis.

Clearly the above results show a shift with respect to the LEGLOAD results to the effect that the circular leg results are lower and the truss leg results are higher.

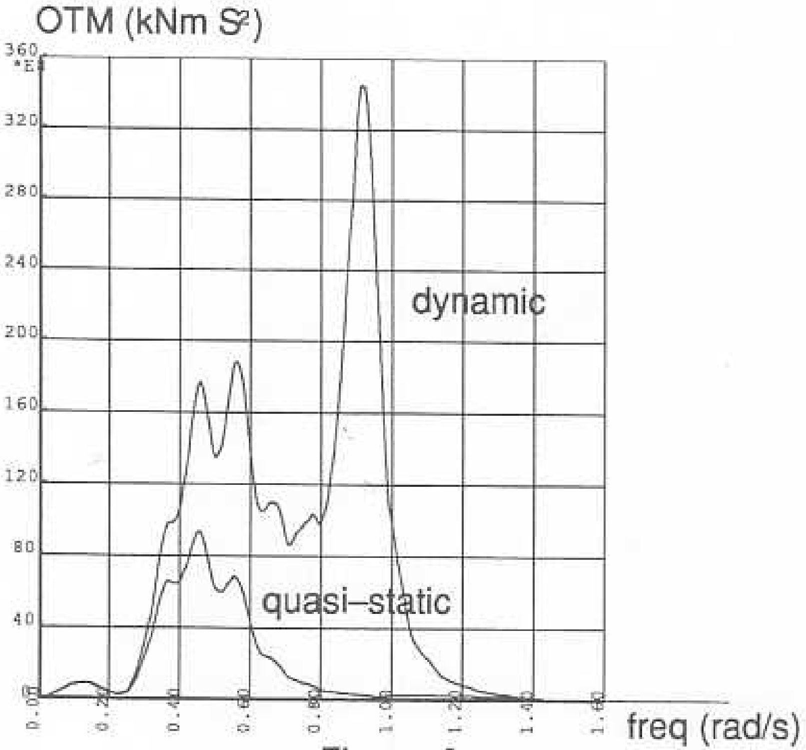


Figure 6.:
Static and dynamic OTM response spectra for the circular leg.

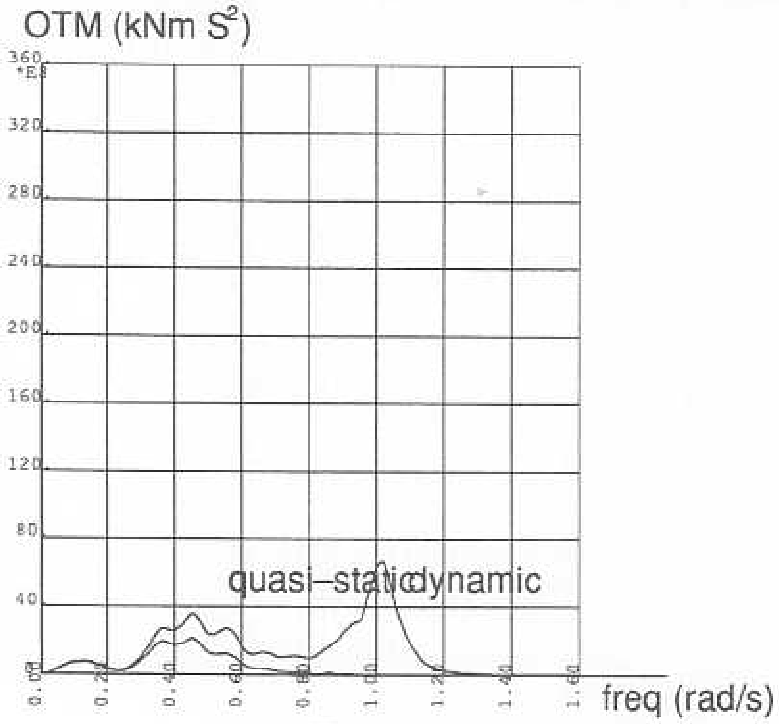


Figure 7.:
Static and dynamic OTM response spectra for the truss leg.

If we repeat the cost estimate for these results we find the following:

| | *circular leg* | | *truss leg* | |
|---|---|---|---|---|
| | *ton* | *cost Dfl x $10^6$* | *ton* | *cost Dfl x $10^6$* |
| Legs (excl. footing) | 1450 | 8.7 | 1115 | 15.6 |
| Footings (extra) | 30 | 0.4 | – | – |
| Jackhouses (extra) | 40 | 0.4 | – | – |
| Hull (extra) | 90 | 0.6 | – | – |
| Jacking system (extra) | – | 0.4 | – | –.– |
| Totals | 1610 | 10.5 | 1115 | 15.6 |
| Extra weight | 495 | – | – | – |
| Extra cost | – | – | – | 5.1 |

Table 8.:
Cost difference based on SIMSEP analysis.

From this it follows that basic design decision to be made between the two leg types need not be changed and that in fact the SIMSEP analysis shows that the structural differences between the two are less pronounced and that consequently the cost advantage of the circular leg unit is increased by 75%.

## DISCUSSION

Going back to the straight comparison of the end results of both types of analysis we find that the most significant difference in results is the reduction of values for the cylindrical leg and an increase of values for the truss leg.

The first explanation for this is the increased drag coefficient for the random time domain analysis. Due to the large influence of the inertia (wave) forces in case of the circular leg the increase of drag coefficients only, causes a relatively higher increase of the total (wave) force for the truss leg.

The next possible cause of this shift could be in the dynamic behaviour. As discussed before the relative contribution from drag and inertia forces are drastically different for both leg types.

The single mass–spring analysis, as used to calculate the DAF in the quasi–static analysis, is completely insensitive to this.

Ref [2] suggests that for a drag dominated wave force the 'classical DAF' would need to be reduced by a factor of 8/3π (0.85). Such a reduction would lead to a reduced dynamic effect in the quasi–static analysis of the truss leg and as such would widen the gap between the quasi–static and the time domain analysis.

To investigate this further we can compare the static results for both leg– and analysis types.

To investigate the dynamic behaviour we define the following DAF:

$$DAF = \frac{max._{dyn} - mean_{dyn}}{max._{stat} - mean_{stat}}$$

This DAF is determined for three main response parameters: the hull deflection (DEFL), total base shear (BS) and total overturning moment (OTM) from both the quasi–static and the time domain results.

This provides the following result:

| | *circular leg* | *truss leg* |
|---|---|---|
| Quasi–static (LEGLOAD) | | |
| DEFL | 1.31 | 1.24 |
| OTM | 1.42 | 1.31 |
| BS | 1.12 | 1.10 |
| Time Domain (SIMSEP) | | |
| DEFL | 1.24 | 1.41 |
| OTM | 1.32 | 1.52 |
| BS | 1.11 | 1.27 |

Table 9.
DAF's from quasi–static and irregular time domain analysis.

These ratios show the same trend as the maxima reported in tables 4 and 7, reduction for the circular and increase for the truss leg.

For further illustration we also analysed both leg alternatives with SIMSEP but with a regular design wave instead of the irregular wave definition.

Figure 8 shows the forcing function and the deflection response for the circular leg. This shows that the structure is definitely not equivalent to a single mass–spring system. The behaviour of the truss leg is similar.

DAF results for this analysis are as follows:

| | *circular leg* | *truss leg* |
|---|---|---|
| Time Domain Regular (SIMSEP) | | |
| DEFL | 2.60 | 1.83 |
| OTM | 2.90 | 1.98 |
| BS | 2.08 | 1.58 |

Table 10.:
DAF's from a regular time domain analysis.

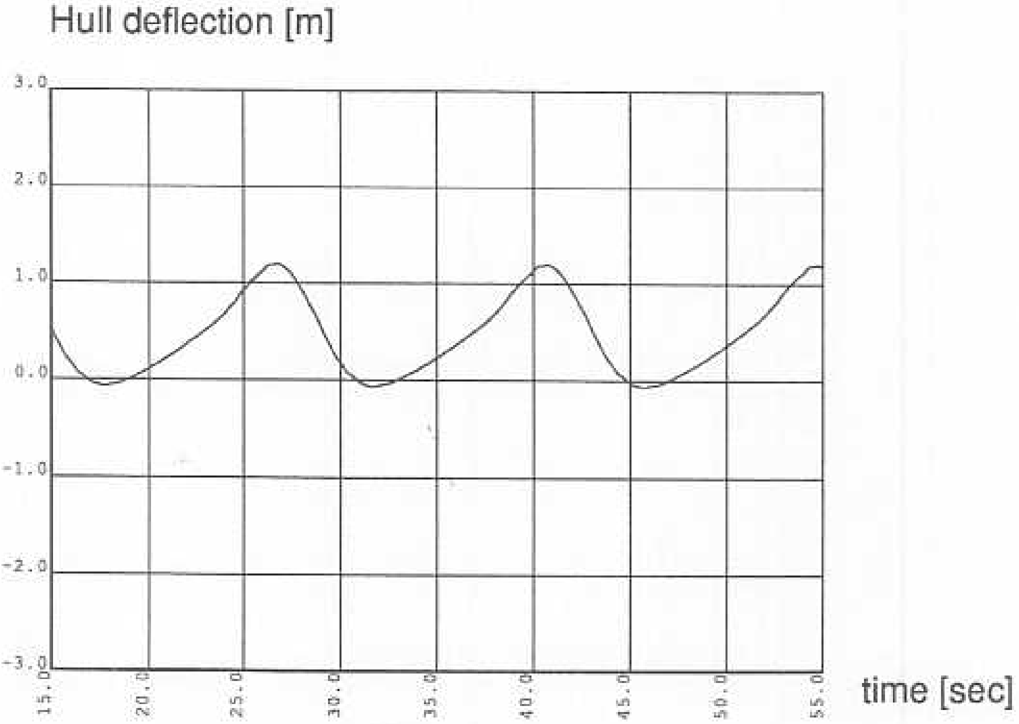


Figure 8.a.:
Static hull deflection for the circular leg.

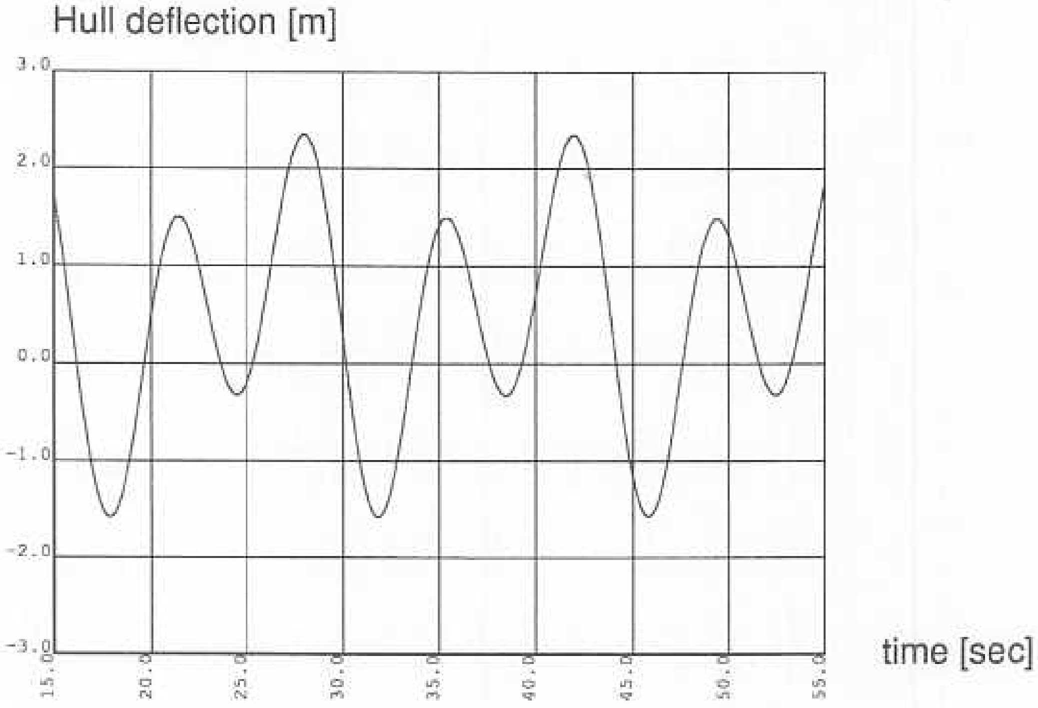


Figure 8.b.:
Dynamic hull deflection response for the circular leg.

The results from tables 9 and 10 show that the differences in maxima between both analysis methods cannot be explained from dynamics alone.

The quasi–static method with the single mass analogy DAF is obviously not strictly correct since the structure is a multi mass spring system and the forcing function is definitely not sinusoidal.

The regular wave method uses the correct structural definition but the regular design wave is far from realistic and is very sensitive to resonance with other harmonies from wave theories and/or leg position etc.

Obviously the difference in maxima is due to the interaction of the irregularity and amount of non–linearity of the wave (forces) and the susceptibility to the dynamic excitation.

This effect is elegantly demonstrated by a method as outlined in ref [3].

Figure 9 shows the probability of non–exceedance of OTM peaks for both the cylindrical and the truss leg taken from the 10 3 hrs simulation. The peak values are normalized with respect to the mean value as follows:

$$P_n = \frac{peak - mean}{standard\ deviation}$$

such that a peak value can be determined as:

$$Peak = mean + P_n \times standard\ deviation$$

Also shown in this figure are curves for a Morison force on a single element in an irregular sea for two additional ratios of drag/ inertia K, $K = \infty$ (100% drag, no inertia) and $K = 0$ (no drag, 100% inertia). For a probability of exceedance of 0.999 and $K = \infty$, $P_n \approx 8$ and for $K = 0$, $P_n \approx 3.7$.

Figure 9 shows that the curves for the circular and the truss leg nicely fit between these two extremes. The truss leg curves close to the K = ∞ curve due to its high drag (non linear) content in the wave force.

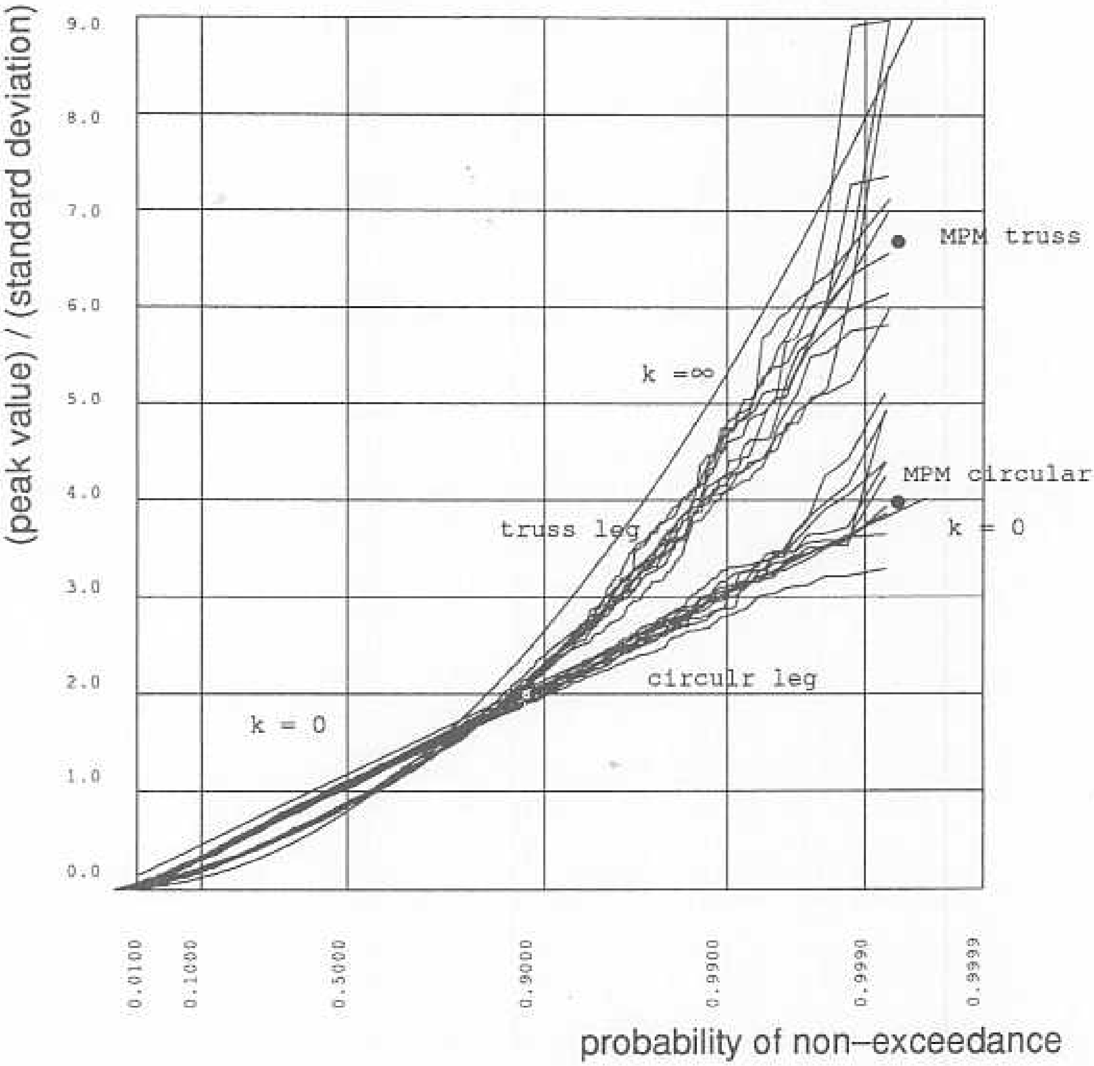


Figure 9.:
Probability of non–exceedance of normalized OTM.

The circular leg curves close to the K = 0 curve due to the high inertia (linear) content in the wave force and due to its higher susceptibility to dynamic (linear) excitation (larger leg added mass).

Following this method the $P_n$ factors can be calculated from the statistical data from the time domain analysis for various response parameters for both the circular and the truss leg.

This yields the following:

| | | *circular leg* | | | | *truss leg* | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *mpm* | *mean* | *stdev* | $P_n$ | *mpm* | *mean* | *stdev* | $P_n$ |
| DEFL | m | 1.31 | .345 | .241 | 4 | 1.06 | .331 | .108 | 6.75 |
| OTM | MNm | 611 | 175 | 109 | 4 | 502 | 169 | 49.6 | 6.71 |
| BS | MN | 10.9 | 2.56 | 2.12 | 3.93 | 8.6 | 2.45 | .91 | 6.76 |
| max leg reaction | MN | 35.9 | 24.1 | 2.9 | 4.07 | 32.6 | 23.6 | 1.34 | 6.74 |

Table 11.
$P_n$ factors for circular and truss leg.

The circular leg shows a remarkable consistent $P_n$ factor of 4 (+/– 1.7%) and the truss leg a factor of 6.74 (+/– 3%).

Table 11 also shows that the means from the circular and truss are quite close.

This is to be expected since both are subjected to the same current and wind loading and the CD x D factors are quite close.

The difference in MPM value is therefore caused by the difference in the standard deviation and the $P_n$ factor. Generally the standard deviation for the circular leg is 2.23 (+/– 4%) times the standard deviation for the truss leg.

The $P_n$ factor shows an opposite behaviour. $P_n$ circular is 0.59 $P_n$ truss.

This leads to a straightforward generally valid definition of the relation between the circular and truss time domain results as:

$$MPM_{circ} = 1.31\,MPM_{truss} - 0.31\,MEAN$$

Above comparisons lead us to conclude that the differences in results between the quasi–static and the time domain results can be attributed to a combination of factors:

- the quasi–static method with single mass spring analogy DAF is not properly accounting for effects of variation of the drag / inertia ratios in the wave forces
- the 1.4 factor on the CD component wave force is a direct cause for a shift in results between the two analyses

## CONCLUSIONS

The application of two essentially different analysis methods (non–linear quasi–static + single degree of freedom DAF and stochastic, non–linear, time domain) to one particular design problem shows that:

- Both methods in this case lead to the same design decision

- The results from the simple quasi–static method are within +/– 10% from the time domain results

- Analysis of the differences between the two methods shows that inclusion of DAF through single mass–spring analogy is inadequate in accounting for dynamic effects from non–linearity in the loading. In this context it seems that the results of the quasi–static + DAF method is only within an +/– 10% band of deviation from the time domain results because the DAF in absolute terms is approx 10%

- The ratio of drag and inertia load on the leg has a significant influence on the end results. In this context the 1.4 factor on the drag coefficient to account for differences in wave kinematics for the various wave theories seems very coarse

- The use of a stochastic non–linear time domain analysis although more time consuming than the quick quasi–static method, has proven to be useful in confirming and fine–tuning design decisions

- Reviewing the results of the analysis of the relative small jack–up design for the Southern North Sea in max. 45 m (150 ft) water depth leads us to conclude that the necessity or practicality of using stochastic non–linear time domain analyses is not limited to deep water applications

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